**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:** **B**

Let time taken for service transmission = X

X is normally distributed with mean µ = 45 minutes and standard deviation σ = 8 minutes

Time delay = 10 minutes

Available time to finish the work = 60 - 10 = 50 minutes

Formula to find Z value is Z = (X - µ) / σ

First, we have to find probability of finishing work in time i.e. meet the commitment, which is,

P(X≤50) = P(Z≤((50-45)/8)) = P(Z≤(5/8)) = P(Z≤0.625) = 0.7324 (From Z Distribution Table)

Therefore, P(X>50) = 1 - P(X≤50) = 1 – 0.7324 = **0.2676**

If we calculate this in Python,







1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Ans:** False

We have normal distribution with mean *μ* = 38 and standard deviation *σ* = 6.

Probability of employee greater than 44 = P(X>44), which is,

P(X>44) = 1 – P(X≤44)

Z = (X - µ) / σ = (X – 38) / 6

Therefore, P(X>44) = 1 - P(Z≤(44 – 38) / 6) P(Z≤1) = 1 - 0.8413 (From Z Distribution Table) = 0.1587 = 15.87

Therefore, above statement is false.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:** True

Z = (X - µ) / σ

P(X≤30) = P(Z≤(30 – 38) / 6) = P(Z≤-1.33) = 0.918 (From Z distribution table)

Expected count = 400 \* 0.918 = .3672 = 36.72%

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:** 2 *X*1 is simply a larger scale version of random variable *X1*. If *X1* is normally distributed then 2 *X*1 is also normally distributed.

Both *X*1 and *X*2 are normally distributed. The associated sums and random samples are also normal, with the appropriate parameters.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:** D

Here, we need range of 99% data which lies between 3rd standard deviation of the mean.

Here, μ = 100, σ = 20.

From empirical rule, μ ± 3σ = 100 ± 3 \* 20 = 100 ± 60 → (40, 160)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:** Let, X is the sum of two random variables having normal distribution.

E(X) = E(profit1 + profit2) = E(5 + 7) = $ 12 Million

Converting to Rupee → 12 \* 45 = 540 Million

SD(X) = SD(profit1 + profit2) = 45 \* ()

= 45 \* = 45 \* 5 = 225 Million Rupees

Therefore, **X~N(540, 2252)**

1. From the empirical rule, approximately 95% of the data falls within two standard deviation of the mean.

μ ± 2σ = 540 ± 2\*225 = 540 ± 450 → **(90, 990)**

1. We can calculate 5th percentile by using formula,

μ – 1.5σ = 540 – (1.5 \* 225) = 540 – 337.5 = 202.5 Million Rupees

1. For division1, Z score for a profit of zero: Z = (X - µ) / σ = (0 – 7) / 4 = -1.75 = 0.0401.

Therefore, division2 has a higher profitability of making a loss.